Superstrings, Geometry, Topology, and C*-algebras

This volume consists of ten articles which provide an in-depth and reader-friendly survey of some of the foundational aspects of singularity theory. Authored by world experts, the various contributions deal with both classical material and modern developments, covering a wide range of topics which are linked to each other in fundamental ways. Singularities are ubiquitous in mathematics and science in general. Singularity theory interacts energetically with the rest of mathematics, acting as a crucible where different types of mathematical problems interact, surprising connections are born and simple questions lead to ideas which resonate in other parts of the subject. This is the first volume in a series which aims to provide an accessible account of the state-of-the-art of the subject, its frontiers, and its interactions with other areas of research. The book is addressed to graduate students and newcomers to the theory, as well as to specialists who can use it as a guidebook.

Analysis, Geometry and Topology of Elliptic Operators

Differential Geometry and Topology, Discrete and Computational Geometry

This volume presents an array of topics that introduce the reader to key ideas in active areas in geometry and topology. The material is presented in a way that both graduate students and researchers should find accessible and enticing. The topics covered range from Morse theory and complex geometry theory to geometric group theory, and are accompanied by exercises that are designed to deepen the reader's understanding and to guide them in exciting directions for future investigation.

Applications of Contact Geometry and Topology in Physics

Three-dimensional Geometry and Topology
Modern theory of elliptic operators, or simply elliptic theory, has been shaped by the Atiyah-Singer Index Theorem created 40 years ago. Reviewing elliptic theory over a broad range, 32 leading scientists from 14 different countries present recent developments in topology; heat kernel techniques; spectral invariants and cutting and pasting; noncommutative geometry; and theoretical particle, string and membrane physics, and Hamiltonian dynamics. The first of its kind, this volume is ideally suited to graduate students and researchers interested in careful expositions of newly-evolved achievements and perspectives in elliptic theory. The contributions are based on lectures presented at a workshop acknowledging Krzysztof P Wojciechowski's work in the theory of elliptic operators.

**A First Course in Geometric Topology and Differential Geometry**

This book is an introduction to several active research topics in Foliation Theory and its connections with other areas. It contains expository lectures showing the diversity of ideas and methods converging in the study of foliations. The lectures by Aziz El Kacimi Alaoui provide an introduction to Foliation Theory with emphasis on examples and transverse structures. Steven Hurder's lectures apply ideas from smooth dynamical systems to develop useful concepts in the study of foliations: limit sets and cycles for leaves, leafwise geodesic flow, transverse exponents, Pesin Theory and hyperbolic, parabolic and elliptic types of foliations. The lectures by Masayuki Asaoka compute the leafwise cohomology of foliations given by actions of Lie groups, and apply it to describe deformation of those actions. In his lectures, Ken Richardson studies the properties of transverse Dirac operators for Riemannian foliations and compact Lie group actions, and explains a recently proved index formula. Besides students and researchers of Foliation Theory, this book will be interesting for mathematicians interested in the applications to foliations of subjects like Topology of Manifolds, Differential Geometry, Dynamics, Cohomology or Global Analysis.

**Geometric topology**

Differential geometry and topology have become essential tools for many theoretical physicists. In particular, they are indispensable in theoretical studies of condensed matter physics, gravity, and particle physics. Geometry, Topology and Physics, Second Edition introduces the ideas and techniques of differential geometry and topology at a level suitable for postgraduate students and researchers in these fields. The second edition of this popular and established text incorporates a number of changes designed to meet the needs of the reader and reflect the development of the subject. The book features a considerably expanded first chapter, reviewing aspects of path integral quantization and gauge theories. Chapter 2 introduces the mathematical concepts of maps, vector spaces, and topology. The following chapters focus on more elaborate concepts in geometry and topology and discuss the application of these concepts to liquid crystals, superfluid helium, general relativity, and bosonic string theory. Later chapters unify geometry and topology, exploring fiber bundles, characteristic classes, and index theorems. New to this second edition is the proof of the index theorem in terms of supersymmetric quantum mechanics. The final two chapters are devoted to the most fascinating applications of geometry and topology in contemporary physics, namely the study of anomalies in gauge field theories and the analysis of Polakov's bosonic string theory from the geometrical point of view. Geometry, Topology and Physics, Second Edition is an ideal introduction to differential geometry and topology for postgraduate students and researchers in theoretical and mathematical physics.

**Foliations: Dynamics, Geometry and Topology**

Applications from condensed matter physics, statistical mechanics and elementary particle theory appear in the book. An obvious omission here is general relativity--we apologize for this. We originally intended to discuss general relativity. However, both the need to keep the size of the book within the reasonable limits and the fact that accounts of the topology and geometry of relativity are already available, for example, in The Large Scale Structure of Space-Time by S. Hawking and G. Ellis, made us reluctantly decide to omit this topic.
**Differential Geometry and Topology**

In 1993, a conference was held honouring mathematician Raoul Bott on his 70th birthday. The lectures given at this conference, along with other important mathematical contributions, are presented in this volume in honour of Raoul Bott.

**Handbook of Geometry and Topology of Singularities I**


**Geometry & Topology**

Although contact geometry and topology is briefly discussed in V I Arnol'd's book "Mathematical Methods of Classical Mechanics" (Springer-Verlag, 1989, 2nd edition), it still remains a domain of research in pure mathematics, e.g. see the recent monograph by H Geiges "An Introduction to Contact Topology" (Cambridge U Press, 2008). Some attempts to use contact geometry in physics were made in the monograph "Contact Geometry and Nonlinear Differential Equations" (Cambridge U Press, 2007). Unfortunately, even the excellent style of this monograph is not sufficient to attract the attention of the physics community to this type of problems. This book is the first serious attempt to change the existing status quo. In it we demonstrate that, in fact, all branches of theoretical physics can be rewritten in the language of contact geometry and topology: from mechanics, thermodynamics and electrodynamics to optics, gauge fields and gravity; from physics of liquid crystals to quantum mechanics and quantum computers, etc. The book is written in the style of famous Landau–Lifshitz (L–L) multivolume course in theoretical physics. This means that its readers are expected to have solid background in theoretical physics (at least at the level of the L–L course). No prior knowledge of specialized mathematics is required. All needed new mathematics is given in the context of discussed physical problems. As in the L–L course some problems/exercises are formulated along the way and, again as in the L–L course, these are always supplemented by either solutions or by hints (with exact references). Unlike the L–L course, though, some definitions, theorems, and remarks are also presented. This is done with the purpose of stimulating the interest of our readers in deeper study of subject matters discussed in the text.


Readership: Students in applied mathematics and theoretical physics. Keywords: Force-Free Fields; Contact and Sub-
Riemannian Geometry; Optimal Control; Theoretical Physics

Key Features: This book is the world’s first book on contact/sub-Riemannian geometry and topology for physicists. Unlike books discussing mathematical methods for physicists, this book discusses physical problems first and only then uses new mathematics to solve these problems. Problems are selected from practically all branches of theoretical physics. This is done with the purpose of demonstrating that contact geometry should be looked upon as a universal language/technical tool of theoretical physics.

Reviews: “This book is written in the style of the well-known Landau-Lifshitz multivolume course in theoretical physics and its prime goal, as the author puts it, is to show the diversity of applications of contact geometry and topology. I enjoyed reading this book, in which the author allows readers to see for themselves “the same forest behind different kinds of trees”. I strongly recommend this book to interested readers.” MathSciNet

**Geometry, Topology and Physics**

Leading experts present a unique, invaluable introduction to the study of the geometry and topology of fluid flows. From basic motions on curves and surfaces to the recent developments in knots and links, the reader is gradually led to explore the fascinating world of geometric and topological fluid mechanics. Geodesics and chaotic orbits, magnetic knots and vortex links, continual flows and singularities become alive with more than 160 figures and examples. In the opening article, H. K. Moffatt sets the pace, proposing eight outstanding problems for the 21st century. The book goes on to provide concepts and techniques for tackling these and many other interesting open problems.

**Geometry, Topology and Physics, Second Edition**

Geometric Topology is a foundational component of modern mathematics, involving the study of spacial properties and invariants of familiar objects such as manifolds and complexes. This volume, which is intended both as an introduction to the subject and as a wide ranging resource for those already grounded in it, consists of 21 expository surveys written by leading experts and covering active areas of current research. They provide the reader with an up-to-date overview of this flourishing branch of mathematics.

**Algebraic and Geometric Topology**

Ten high-quality survey articles provide an overview of important recent developments in the mathematics surrounding negative curvature.

**Real Algebraic Geometry and Topology**

Contains sections on Algebraic $K$- and $L$-theory, Surgery and its applications, Group actions.

**Solitons, Geometry, and Topology: On the Crossroad**

This volume contains the proceedings of the 2016 AMS von Neumann Symposium on Topological Recursion and its Influence in Analysis, Geometry, and Topology, which was held from July 4–8, 2016, at the Hilton Charlotte University Place, Charlotte, North Carolina. The papers contained in the volume present a snapshot of rapid and rich developments in the emerging research field known as topological recursion. It has its origin around 2004 in random matrix theory and also in Mirzakhani’s work on the volume of moduli spaces of hyperbolic surfaces. Topological recursion has played a fundamental role in connecting seemingly unrelated areas of mathematics such as matrix models, enumeration of Hurwitz numbers and Gromov-Witten’s dessins d’enfants, Gromov-Witten invariants, the A-polynomials and colored polynomial invariants of knots, WKB analysis, and quantization of Hitchin moduli spaces. In addition to establishing these topics, the volume includes survey papers on the most recent key accomplishments: discovery of the unexpected relation to semi-simple cohomological field theories and a solution to the remodeling conjecture. It also provides a glimpse into the future research direction; for example, connections with the Airy structures, modular functors, Hurwitz-Frobenius manifolds, and
ELSV-type formulas.

**Topology and Geometry for Physicists**

This book contains expository papers that give an up-to-date account of recent developments and open problems in the geometry and topology of manifolds, along with several research articles that present new results appearing in published form for the first time. The unifying theme is the problem of understanding manifolds in low dimensions, notably in dimensions three and four, and the techniques include algebraic topology, surgery theory, Donaldson and Seiberg-Witten gauge theory, Heegaard Floer homology, contact and symplectic geometry, and Gromov-Witten invariants. The articles collected for this volume were contributed by participants of the Conference "Geometry and Topology of Manifolds" held at McMaster University on May 14-18, 2004 and are representative of the many excellent talks delivered at the conference.

**Geometry and Topology Down Under**

§1. Historical Remarks Convex Integration theory, first introduced by M. Gromov [17], is one of three general methods in immersion-theoretic topology for solving a broad range of problems in geometry and topology. The other methods are: (i) Removal of Singularities, introduced by M. Gromov and Y. Eliashberg [8]; (ii) the covering homotopy method which, following M. Gromov's thesis [16], is also referred to as the method of sheaves. The covering homotopy method is due originally to S. Smale [36] who proved a crucial covering homotopy result in order to solve the classification problem for immersions of spheres in Euclidean space. These general methods are not linearly related in the sense that successive methods subsumed the previous methods. Each method has its own distinct foundation, based on an independent geometrical or analytical insight. Consequently, each method has a range of applications to problems in topology that are best suited to its particular insight. For example, a distinguishing feature of Convex Integration theory is that it applies to solve closed relations in jet spaces, including certain general classes of underdetermined non-linear systems of partial differential equations. As a case of interest, the Nash-Kuiper Cl-isometric immersion theorem can be reformulated and proved using Convex Integration theory (cf. Gromov [18]). No such results on closed relations in jet spaces can be proved by means of the other two methods.

**Convex Integration Theory**

This book contains the proceedings of the Real Algebraic Geometry-Topology Conference, held at Michigan State University in December 1993. Presented here are recent results and discussions of new ideas pertaining to such topics as resolution theorems, algebraic structures, topology of nonsingular real algebraic sets, and the distribution of real algebraic sets in projective space.

**Handbook of Geometric Topology**

The uniqueness of this text in combining geometric topology and differential geometry lies in its unifying thread: the notion of a surface. With numerous illustrations, exercises and examples, the student comes to understand the relationship of the modern abstract approach to geometric intuition. The text is kept at a concrete level, avoiding unnecessary abstractions, yet never sacrificing mathematical rigor. The book includes topics not usually found in a single book at this level.

**Geometry, Topology, and Dynamics in Negative Curvature**

Differential geometry and topology have become essential tools for many theoretical physicists. In particular, they are indispensable in theoretical studies of condensed matter physics, gravity, and particle physics. Geometry, Topology and Physics, Second Edition introduces the ideas and techniques of differential geometry and topology at a level suitable for postgraduate students and researchers in these fields. The second edition of this popular and established text incorporates a number of changes
designed to meet the needs of the reader and reflect the development of the subject. The book features a considerably expanded first chapter, reviewing aspects of path integral quantization and gauge theories. Chapter 2 introduces the mathematical concepts of maps, vector spaces, and topology. The following chapters focus on more elaborate concepts in geometry and topology and discuss the application of these concepts to liquid crystals, superfluid helium, general relativity, and bosonic string theory. Later chapters unify geometry and topology, exploring fiber bundles, characteristic classes, and index theorems. New to this second edition is the proof of the index theorem in terms of supersymmetric quantum mechanics. The final two chapters are devoted to the most fascinating applications of geometry and topology in contemporary physics, namely the study of anomalies in gauge field theories and the analysis of Polakov's bosonic string theory from the geometrical point of view. Geometry, Topology and Physics, Second Edition is an ideal introduction to differential geometry and topology for postgraduate students and researchers in theoretical and mathematical physics.

**Geometry and Topology of Submanifolds X**

This book discusses topics ranging from traditional areas of topology, such as knot theory and the topology of manifolds, to areas such as differential and algebraic geometry. It also discusses other topics such as three-manifolds, group actions, and algebraic varieties.

**Handbook of Geometry and Topology of Singularities II**

The book is a collection of surveys and original research articles concentrating on new perspectives and research directions at the crossroads of algebraic geometry, topology, and singularity theory. The papers, written by leading researchers working on various topics of the above fields, are the outcome of the "Némethi60: Geometry and Topology of Singularities" conference held at the Alfréd Rényi Institute of Mathematics in Budapest, from May 27 to 31, 2019. Both the conference and this resulting volume are in honor of Professor András Némethi, on the occasion of his 60th birthday, whose work plays a decisive and influential role in the interactions between the above fields. The book should serve as a valuable resource for graduate students and researchers to deepen the new perspectives, methods, and connections between geometry and topology regarding singularities.

**Techniques of Geometric Topology**

Like any books on a subject as vast as this, this book has to have a point-of-view to guide the selection of topics. Naber takes the view that the rekindled interest that mathematics and physics have shown in each other of late should be fostered, and that this is best accomplished by allowing them to cohabit. The book weaves together rudimentary notions from the classical gauge theory of physics with the topological and geometrical concepts that became the mathematical models of these notions. The reader is asked to join the author on some vague notion of what an electromagnetic field might be, to be willing to accept a few of the more elementary pronouncements of quantum mechanics, and to have a solid background in real analysis and linear algebra and some of the vocabulary of modern algebra. In return, the book offers an excursion that begins with the definition of a topological space and finds its way eventually to the moduli space of anti-self-dual SU(2) connections on S4 with instanton number -1.

**Geometry & Topology**

§1. Historical Remarks Convex Integration theory, rst introduced by M. Gromov [17], is one of three general methods in immersion-theoretic topology for solving a broad range of problems in geometry and topology. The other methods are: (i) Removal of Singularities, introduced by M. Gromov and Y. Eliashberg [8]; (ii) the covering homotopy method which, following M. Gromov's thesis [16], is also referred to as the method of sheaves. The covering homotopy method is due originally to S. Smale [36] who proved a crucial covering homotopy result in order to solve the classificaton problem for immersions of spheres in Euclidean space. These general methods are not linearly related in the sense that successive methods subsumed the previous methods. Each method has its own distinct foundation,
based on an independent geometrical or analytical insight. Consequently, each method has a range of applications to problems in topology that are best suited to its particular insight. For example, a distinguishing feature of Convex Integration theory that is applied to the study of relations in jets, including certain general classes of underdetermined non-linear systems of partial differential equations. As a case of interest, the Nash-Kuiper $C^1$-isometric immersion theorem can be reformulated and proved using Convex Integration theory (cf. Gromov [18]). No such results on closed relations in jet spaces can be proved by means of the other two methods. On the other hand, many classical results in immersion-theoretic topology, such as the classification of immersions, are provable by all three methods.

**An Introduction to the Geometry and Topology of Fluid Flows**

This volume contains the proceedings of an NSF-CBMS Conference held at Texas Christian University in Fort Worth, Texas, May 18-22, 2009. The papers, written especially for this volume by well-known mathematicians and mathematical physicists, are an outgrowth of the talks presented at the conference. Topics examined are highly interdisciplinary and include, among many other things, recent results on D-brane charges in $K$-homology and twisted $K$-homology, Yang-Mills gauge theory and connections with non-commutative geometry, Landau-Ginzburg models, $C^*$-algebraic non-commutative geometry and ties to quantum physics and topology, the rational homotopy type of the group of unitary elements in an Azumaya algebra, and functoriality properties in the theory of $C^*$-crossed products and fixed point algebras for proper actions. An introduction, written by Jonathan Rosenberg, provides an instructive overview describing common themes and how the various papers in the volume are interrelated and fit together. The rich diversity of papers appearing in the volume demonstrates the current interplay between superstring theory, geometry/topology, and non-commutative geometry. The book will be of interest to graduate students, mathematicians, mathematical physicists, and researchers working in these areas.

**Topological Recursion and its Influence in Analysis, Geometry, and Topology**

This is Part 2 of a two-part volume reflecting the proceedings of the 1993 Georgia International Topology Conference held at the University of Georgia during the month of August. The texts include research and expository articles and problem sets. The conference covered a wide variety of topics in geometric topology. Features: Kirby's problem list, which contains a thorough description of the progress made on each of the problems and includes a very complete bibliography, makes the work useful for specialists and non-specialists who want to learn about the progress made in many areas of topology. This list may serve as a reference work for decades to come. Gabai's problem list, which focuses on foliations and laminations of 3-manifolds, collects for the first time in one paper definitions, results, and problems that may serve as a defining source in the subject area.

**Invitations to Geometry and Topology**

This volume contains the courses and lectures given during the workshop on Differential Geometry and Topology held at Alghero, Italy, in June 1992. The main goal of this meeting was to offer an introduction in attractive areas of current research and to discuss some recent important achievements in both the fields. This is reflected in the present book which contains some introductory texts together with more specialized contributions. The topics covered in this volume include circle and sphere packings, 3-manifolds invariants and combinatorial presentations of manifolds, soliton theory and its applications in differential geometry, G-manifolds of low cohomogeneity, exotic differentiable structures on $R^4$, conformal deformation of Riemannian manifolds and Riemannian geometry of algebraic manifolds. Contents: Asymptotic G-Manifolds (A Alekseevsky & D Alekseevsky) Les Paquets de Cercles (M Berger) Smooth Structures in Euclidean Spaces (S Demichelis) Surface Theory, Harmonic Maps and Commuting Hamiltonian Flows (D Ferus) Metric Invariants of Kähler Manifolds (M Gromov) On the Sphere Packing Problem and the Proof of Kepler's Conjecture (W Y Hsiang) A 3-Gem Approach to Turaev-Viro Invariants (S L S Lins) Cohomology Operations and Modular Invariant Theory (L
Stacks and Categories in Geometry, Topology, and Algebra

This volume on pure and applied differential geometry, includes topics on submanifold theory, affine differential geometry and applications of geometry in engineering sciences. The conference was dedicated to the 70th birthday of Prof Katsumi Nomizu. Papers on the scientific work and life of Katsumi Nomizu are also included.

Geometry and Topology


Issues in Algebra, Geometry, and Topology / 2013 Edition is a ScholarlyEditions™ book that delivers timely, authoritative, and comprehensive information about Topology. The editors have built Issues in Algebra, Geometry, and Topology: 2013 Edition on the vast information databases of ScholarlyNews.™ You can expect the information about Topology in this book to be deeper than what you can access anywhere else, as well as consistently reliable, authoritative, informed, and relevant. The content of Issues in Algebra, Geometry, and Topology: 2013 Edition has been produced by the world’s leading scientists, engineers, analysts, research institutions, and companies. All of the content is from peer-reviewed sources, and all of it is written, assembled, and edited by the editors at ScholarlyEditions™ and available exclusively from us. You now have a source you can cite with authority, confidence, and credibility. More information is available at http://www.ScholarlyEditions.com/.

Topology and Geometry for Physics

This book contains the proceedings of the conference Geometry & Topology Down Under, held July 11-22, 2011, at the University of Melbourne, Parkville, Australia, in honour of Hyam Rubinstein. The main topic of the book is low-dimensional geometry and topology. It includes both survey articles based on courses presented at the conferences and research articles devoted to important questions in low-dimensional geometry. Together, these contributions show how methods from different fields of mathematics contribute to the study of 3-manifolds and Gromov hyperbolic groups. It also contains a list of favorite problems by Hyam Rubinstein.

Convex Integration Theory


Geometry And Topology Of Submanifolds Ix

Fully refereed international journal dealing with all aspects of geometry and topology and their applications.

Singularities and Their Interaction with Geometry and Low Dimensional Topology

Fully refereed international journal dealing with all aspects of geometry and topology and their applications.

Topology, Geometry and Gauge fields
Every mathematician should be acquainted with the basic facts about the geometry of surfaces, of two-dimensional manifolds. The theory of three-dimensional manifolds is much more difficult and still only partly understood, although there is ample evidence that the theory of three-dimensional manifolds is one of the most beautiful in the whole of mathematics. This excellent introductory work makes this mathematical wonderland remained rather inaccessible to non-specialists. The author is both a leading researcher, with a formidable geometric intuition, and a gifted expositor. His vivid descriptions of what it might be like to live in this or that three-dimensional manifold bring the subject to life. Like Poincaré, he appeals to intuition, but his enthusiasm is infectious and should make many converts for this kind of mathematics. There are good pictures, plenty of exercises and problems, and the reader will find a selection of topics which are not found in the standard repertoire. This book contains a great deal of interesting mathematics.

**Geometry And Topology Of Submanifolds VII: Differential Geometry In Honour Of Prof Katsumi Nomizu**

Contents:Affine Bibliography 1998 (T Binder et al.)Contact Metric R-Harmonic Manifolds (K Arslan & C Murathan)Local Classification of Centroaffine Tchebychev Surfaces with Constant Curvature Metric (T Binder)Hypersurfaces in Space Forms with Some Constant Curvature Functions (F Brito et al.)Some Relations Between a Submanifold and Its Focal Set (S Carter & A West)On Manifolds of Pseudosymmetric Type (F Defever et al.)Hypersurfaces with Pseudosymmetric Weyl Tensor in Conformally Flat Manifolds (R Deszcz et al.)Least-Squares Geometrical Fitting and Minimising Functions on Submanifolds (F Dillen et al.)Cubic Forms Generated by Functions on Projectively Flat Spaces (J Leder)Distinguished Submanifolds of a Sasakian Manifold (I Mihai)On the Curvature of Left Invariant Locally Conformally Para-Kählerian Metrics (Z Olszak)Remarks on Affine Variations on the Ellipsoid (M Wiehe)Dirac's Equation, Schrödinger's Equation and the Geometry of Surfaces (T J Willmore)and other papers

Readership: Researchers doing differential geometry and topology.

Keywords:Proceedings;Geometry;Topology;Valenciennes (France);Lyon (France);Leuven (Belgium);Dedication

**From Geometry to Topology**

**Geometry and Topology of Manifolds**

A concise but self-contained introduction of the central concepts of modern topology and differential geometry on a mathematical level is given specifically with applications in physics in mind. All basic concepts are systematically provided including sketches of the proofs of most statements. Smooth finite-dimensional manifolds, tensor and exterior calculus operating on them, homotopy, (co)homology theory including Morse theory of critical points, as well as the theory of fiber bundles and Riemannian geometry, are treated. Examples from physics comprise topological charges, the topology of periodic boundary conditions for solids, gauge fields, geometric phases in quantum physics and gravitation.

**Geometry, Topology, & Physics for Raoul Bott**

This volume contains the proceedings of the CATS4 Conference on Higher Categorical Structures and their Interactions with Algebraic Geometry, Algebraic Topology and Algebra, held from July 2-7, 2012, at CIRM in Luminy, France. Over the past several years, the CATS conference series has brought together top level researchers from around the world interested in relative and higher category theory and its applications to classical mathematical domains. Included in this volume is a collection of articles covering the applications of categories and stacks to geometry, topology and algebra. Techniques such as localization, model categories, simplicial objects, sheaves of categories, mapping stacks, dg structures, hereditary categories, and derived stacks, are applied to give new insight on cluster algebra, Lagrangians, trace theories, loop spaces, structured surfaces, stability, ind-coherent complexes and
1-affineness showing up in geometric Langlands, branching out to many related topics along the way.